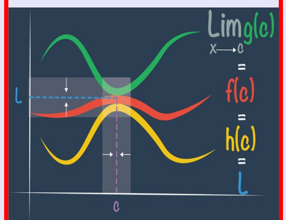
Math 261

Fall 2022

Lecture 17



Differentiation Formula:

1) $\frac{d}{dx}[c] = 0$

2) $\frac{d}{dx}[x] = 1$

3) $\frac{d}{dx}[x^n] = n\,x^{n-1}$ Power Rule

4) $\frac{d}{dx}[c\,f(x)] = c\,\frac{d}{dx}[f(x)]$

5) $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

6) $\frac{d}{dx}[f(x) \cdot g(x)] = \frac{d}{dx}[f(x)] \cdot g(x) + f(x) \cdot \frac{d}{dx}[g(x)]$

Product Rule

7) $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{d}{dx}[f(x)] \cdot g(x) - f(x) \cdot \frac{d}{dx}[g(x)]}{[g(x)]^2}$

Quotient Rule

Find $\frac{d}{dx}\left[(x^3+2)(x^3-2)\right]$

Using Product Rule

$$= \frac{d}{dx}[x^3+2]\cdot(x^3-2) + (x^3+2)\cdot\frac{d}{dx}[x^3-2]$$

$$= (3x^2+0)\cdot(x^3-2) + (x^3+2)\cdot(3x^2-0)$$

$$= 3x^2(x^3-2) + 3x^2(x^3+2)$$

$$= 3x^2[x^3-2+x^3+2] = 3x^2\cdot 2x^3$$

$$= \boxed{6x^5}$$

Another Method

$$\frac{d}{dx}\left[(x^3-2)(x^3+2)\right] = \frac{d}{dx}\left\{(x^3)^2-2^2\right\}$$

$$= \frac{d}{dx}[x^6-4] = 6x^{6-1}-0$$

$$= \boxed{6x^5}$$

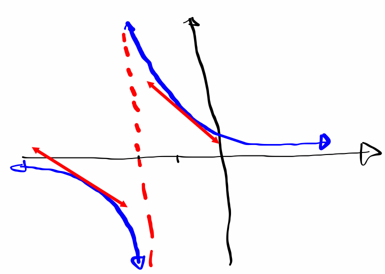
Find $\frac{d}{dx}\left[\frac{1}{x+2}\right]$

Using quotient Rule

$$= \frac{\overset{0}{\frac{d}{dx}[1]}\cdot(x+2) - 1\cdot\overset{1}{\frac{d}{dx}[x+2]}}{(x+2)^2}$$

$$= \frac{0-1\cdot 1}{(x+2)^2} = \frac{-1}{(x+2)^2} < 0$$

Graph $f(x) = \frac{1}{x+2}$



Any tan. line to this graph is a decreasing line $\rightarrow m<0$

$f'(x)<0$

Find $\frac{d}{dx}\left[\frac{4x^2+5}{x}\right] = \frac{\frac{d}{dx}[4x^2+5]\cdot x - (4x^2+5)\cdot\frac{d}{dx}[x]}{(x)^2}$

$$= \frac{8x\cdot x - (4x^2+5)\cdot 1}{x^2}$$

$$= \frac{8x^2-4x^2-5}{x^2} = \boxed{\frac{4x^2-5}{x^2}}$$

Another Method

$$\frac{d}{dx}\left[\frac{4x^2+5}{x}\right] = \frac{d}{dx}\left[\frac{4x^2}{x}+\frac{5}{x}\right]$$

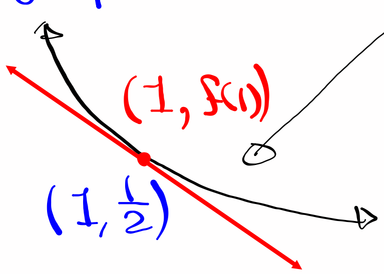
$$= \frac{d}{dx}\left[4x+5x^{-1}\right]$$

$$= \frac{d}{dx}[4x] + \frac{d}{dx}[5x^{-1}] = 4\underbrace{\frac{d}{dx}[x]}_{1} + 5\frac{d}{dx}[x^{-1}]$$

$$= 4\cdot 1 + 5\cdot(-1)x^{-1-1}$$

$$= 4 - 5x^{-2} = \boxed{4-\frac{5}{x^2}}$$

Find the equation of the tan. line to the graph of $f(x) = \frac{x}{x^2+1}$ at $x=1$.



$$f'(x) = \frac{1(x^2+1) - x\cdot 2x}{(x^2+1)^2}$$

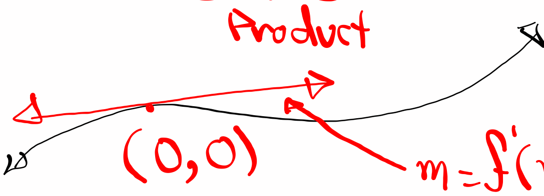
$$m = f'(x)\Big|_{x=1} \qquad = \frac{1-x^2}{(x^2+1)^2}$$

$$m = f'(1) = \frac{1-1^2}{(1^2+1)^2} = 0$$

$$y - y_1 = m(x-x_1)$$

$$y - \frac{1}{2} = 0(x-1) \Rightarrow \boxed{y=\frac{1}{2}}$$

Find equation of tan. line to the graph of $f(x) = \underbrace{x \cos x}_{\text{Product}}$ at $x = 0$.



$m = f'(x)\big|_{x=0}$

$y - y_1 = m(x - x_1)$

$y - 0 = 1(x - 0)$

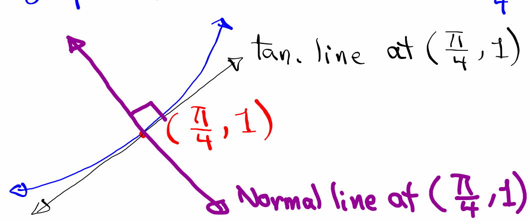
$\boxed{y = x}$

$f'(x) = 1 \cdot \cos x + x \cdot (-\sin x)$

$f'(x) = \cos x - x \sin x$

$f'(0) = \cancel{\cos 0}^{\,1} - \cancel{0 \cdot \sin 0}^{\,0} = 1$

Find equation of the normal line to the graph $f(x) = \tan x$ at $x = \frac{\pi}{4}$.



$m_{\text{Normal line}} = \dfrac{-1}{m_{\text{tan. line}}} = \dfrac{-1}{f'(\frac{\pi}{4})} = \boxed{\dfrac{-1}{2}}$

$f(x) = \tan x$

$f'(x) = \sec^2 x \rightarrow f'(\frac{\pi}{4}) = \sec^2 \frac{\pi}{4} = \dfrac{1}{\cos^2 \frac{\pi}{4}} = \dfrac{1}{(\frac{\sqrt{2}}{2})^2}$

(See notes from Yesterday)

$= \dfrac{1}{\frac{2}{4}} = \dfrac{1}{\frac{1}{2}} = \boxed{2}$

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{-1}{2}(x - \frac{\pi}{4})$

$\boxed{y = \frac{-1}{2}x + \frac{\pi}{8} + 1}$

Prove the quotient Rule

LCD $= g(x+h)\cdot g(x)$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \lim_{h\to 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)\cdot g(x) - f(x)\cdot g(x+h)}{h\, g(x+h)\, g(x)}$$

$$= \lim_{h\to 0} \frac{f(x+h)g(x) - f(x)g(x) - f(x)g(x+h) + f(x)g(x)}{h\, g(x+h)\, g(x)}$$

$$= \lim_{h\to 0} \frac{g(x)[f(x+h) - f(x)] - f(x)[g(x+h) - g(x)]}{h\, g(x+h)\, g(x)}$$

$$= \lim_{h\to 0} \frac{g(x)\boxed{f(x+h) - f(x)}}{\boxed{h}\, g(x+\cancel{h}^{0})\, g(x)} - \lim_{h\to 0} \frac{f(x)\boxed{g(x+h) - g(x)}}{\boxed{h}\, g(x+\cancel{h}^{0})\, g(x)}$$

$$= \frac{g(x)\cdot f'(x)}{[g(x)]^2} - \frac{f(x)\cdot g'(x)}{[g(x)]^2}$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{[g(x)]^2}$$

Given $f(x) = x + \sqrt{x}$

1) Domain: $[0, \infty)$

2) Find $f(4) = 6$

3) Find $f'(4) = 1 + \frac{1}{2\sqrt{4}}$

$= 1 + \frac{1}{4} = \boxed{\frac{5}{4}}$

$f(x) = x + x^{\frac{1}{2}}$

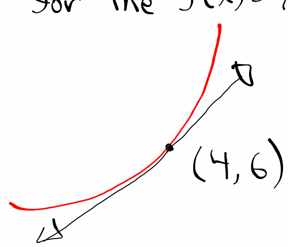
$f'(x) = 1 + \frac{1}{2}x^{\frac{1}{2}-1}$

$f'(x) = 1 + \frac{1}{2}x^{-1/2}$

$= 1 + \frac{1}{2x^{1/2}}$

$= 1 + \frac{1}{2\sqrt{x}}$

4) Find eqn of tan. line at $x=4$ for the $f(x) = x + \sqrt{x}$



$y - y_1 = m(x - x_1)$

$y - 6 = \frac{5}{4}(x - 4)$

$\boxed{y = \frac{5}{4}x + 1}$

Find $\frac{d}{dx}[\cot x]$

$$= \frac{d}{dx}\left[\frac{\cos x}{\sin x}\right] = \frac{\frac{d}{dx}[\cos x]\cdot \sin x - \cos x \cdot \frac{d}{dx}[\sin x]}{\sin^2 x}$$

$$= \frac{-\sin x \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x}$$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-(1)}{\sin^2 x}$$

$$\boxed{\frac{d}{dx}[\cot x] = -\csc^2 x}$$

Find $\frac{d}{dx}[\sec x] = \frac{d}{dx}\left[\frac{1}{\cos x}\right]$

$$= \frac{\cancel{0\cdot\cos x}^{\,0} - 1\cdot(-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x}$$

$$= \frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}$$

$$\boxed{\frac{d}{dx}[\sec x] = \tan x \cdot \sec x}$$